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Comment on "Radiative decays, nonet symmetry and SU(3)  
breaking".

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Abstract

We comment on the paper by Benayoun, DelBuono, Eidelman, Ivanchenko, and O'Connell [Phys. Rev. **D 59**, 114027 (1999)]. We show that the decay  $\phi \rightarrow \pi^0 \gamma$  is absent in model of the nonet symmetry and SU(3) breaking suggested by authors.

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A recent paper [1] reexamined "the problem of simultaneously describing in a consistent way all radiative and leptonic decays of light mesons ( $V \rightarrow P\gamma$ ,  $P \rightarrow V\gamma$ ,  $P \rightarrow \gamma\gamma$ ,  $V \rightarrow e^+e^-$ )". Unfortunately, this reexamination cannot help but provoke objections. The authors use the broken  $U_L(3) \times U_R(3)$  Lagrangian of the hidden local symmetry approach [2] written in terms of ideally mixed  $\omega$  and  $\phi$  states in the following way:

$$L = \dots + \frac{1}{2}af_\pi^2g^2[(\rho_\mu^0)^2 + (\omega_\mu^I)^2 + l_V(\phi_\mu^I)^2] - \quad (1)$$

$$-ae f_\pi^2g[\rho_\mu^0 + \frac{1}{3}\omega_\mu^I + l_V\frac{\sqrt{2}}{3}\phi_\mu^I]A_\mu + \frac{1}{2}C\epsilon_{\mu\nu\rho\delta}\partial_\mu\rho_\nu\partial_\rho\omega_\delta^I\pi^0 + \dots$$

This expression describes the ideally mixed vector mesons [ $\omega^I = (u\bar{u} + d\bar{d})/\sqrt{2}$ ,  $\phi^I = -s\bar{s}$ ], the kinetic and mass terms, their coupling to the electromagnetic field, and hadron states except for the coupling of the  $\phi$  meson to the  $\rho\pi$  states and consequently to  $\gamma\pi^0$  state.

The authors of Ref. [1] wanted to introduce this coupling taking into account a deviation from the ideal mixing of the  $\omega$  and  $\phi$  mesons. As is generally known, see, for example, Ref. [3], to do this in the tree approximation ( just that very approximation was used in Ref. [1] ) the mixing terms should be introduced into the mass and/or kinetic terms of the Lagrangian, then the part of the Lagrangian quadratic in fields should be diagonalized to get physical fields, and then the part of the Lagrangian with interaction should be reexpressed in terms of physical fields. It is the hard-and-fast rule, see also, for example, the diagonalization of axial-vector and pseudoscalar fields or the dynamical and external gauge fields in the hidden local symmetry Lagrangian [2].

Instead the authors of Ref. [1] reexpressed the Lagrangian (1) in terms of "physical" (rotated) fields ( $\omega_\mu$  and  $\phi_\mu$ ) and got the following expression:

$$L = \frac{1}{2}af_\pi^2g^2[(\rho_\mu^0)^2 + (\omega_\mu)^2(\cos^2\delta_V + l_V\sin^2\delta_V) + (\phi_\mu)^2(\sin^2\delta_V + l_V\cos^2\delta_V)] + \quad (2)$$

$$+ (\phi_\mu\omega_\mu)af_\pi^2g^2(l_V - 1)\sin\delta_V\cos\delta_V -$$

$$-ae f_\pi^2g\left[\rho_\mu^0 + \frac{1}{3}\omega_\mu(\cos\delta_V + l_V\sqrt{2}\sin\delta_V) - \frac{1}{3}\phi_\mu(\sin\delta_V - l_V\sqrt{2}\cos\delta_V)\right]A_\mu +$$

$$+ \frac{1}{2}C\cos\delta_V\epsilon^{\mu\nu\rho\delta}\partial_\mu\rho_\nu\partial_\rho\omega_\delta\pi^0 - \frac{1}{2}C\sin\delta_V\epsilon_{\mu\nu\rho\delta}\partial_\mu\rho_\nu\partial_\rho\phi_\delta\pi^0 + \dots,$$

where  $\delta_V$  is an angle which describes the deviation from the ideal mixing angle.

One can see that the last Lagrangian has the nondiagonal square  $\omega_\mu\phi_\mu$  term which describes the  $\omega - \phi$  transitions and, hence, describes the nonphysical  $\omega_\mu$  and  $\phi_\mu$  fields which do not have the definite masses. What fields are physical is decided not by one author or another but by the Lagrangian. In our case the  $\omega_\mu^I$  and  $\phi_\mu^I$  fields are physical.

The authors of Ref. [1] ignored this fact and got formulas dependent on the  $\delta_V$  angle:

$$\begin{aligned} A(\phi \rightarrow \rho\pi \rightarrow \gamma\pi) &= -C \frac{e}{2g} \sin \delta_V, \\ A(\phi \rightarrow \gamma \rightarrow e^+e^-) &= ae f_\pi^2 g \frac{1}{3} \left( l_V \sqrt{2} \cos \delta_V - \sin \delta_V \right) \frac{e}{m_\phi^2}, \end{aligned} \quad (3)$$

and so on. Hereafter we drop the obvious Lorentz structures.

Then they used these formulas and found the mixing parameters from the data. The result is  $\delta_V = -3.33 \pm 0.16^\circ$  and  $l_V = 1.376 \pm 0.031$  which gives the  $\phi$  meson mass equal to  $920 \pm 15$  MeV, see Eq.(1). But, the real trouble is that the  $\delta_V$  dependence is an artifact due to misuse of the Lagrangian (2 ).

It is instructive to show this to the first order in mixing taking into account the nondiagonal  $\omega - \phi$  term in the Lagrangian (2 ):

$$\begin{aligned} A(\phi \rightarrow \gamma\pi) &= A(\phi \rightarrow \rho\pi \rightarrow \gamma\pi) + A(\phi \rightarrow \omega \rightarrow \rho\pi \rightarrow \gamma\pi) = -C \frac{e}{2g} \sin \delta_V + \\ &+ \frac{ea^2 f_\pi^4 g^3 (l_V - 1) \sin \delta_V \cos \delta_V C \cos \delta_V}{2a^2 f_\pi^4 g^4 (\sin^2 \delta_V + l_V \cos^2 \delta_V - \cos^2 \delta_V - l_V \sin^2 \delta_V)} \simeq \\ &-C \frac{e}{2g} \sin \delta_V + \frac{eC(l_V - 1) \sin \delta_V}{2g(l_V - 1)} = 0, \end{aligned} \quad (4)$$

$$\begin{aligned} A(\phi \rightarrow e^+e^-) &= A(\phi \rightarrow \gamma^* \rightarrow e^+e^-) + A(\phi \rightarrow \omega \rightarrow \gamma^* \rightarrow e^+e^-) = \\ &= ae f_\pi^2 g \frac{1}{3} \left( l_V \sqrt{2} \cos \delta_V - \sin \delta_V \right) \frac{e}{m_\phi^2} + \\ &+ \frac{af_\pi^2 g^2 (l_V - 1) \sin \delta_V \cos \delta_V}{af_\pi^2 g^2 (\sin^2 \delta_V + l_V \cos^2 \delta_V - \cos^2 \delta_V - l_V \sin^2 \delta_V)} ae f_\pi^2 g \frac{1}{3} \left( \cos \delta_V + l_V \sqrt{2} \sin \delta_V \right) \frac{e}{m_\phi^2} \simeq \\ &\simeq \frac{ae^2 f_\pi^2 g}{3m_\phi^2} (\sqrt{2} l_V - \sin \delta_V + \frac{(l_V - 1) \sin \delta_V}{(l_V - 1)}) = \frac{\sqrt{2} l_V ae^2 f_\pi^2 g}{3m_\phi^2}, \end{aligned}$$

and so on.

So, all amplitudes, obtained from the Lagrangian (2) to first order in the  $\omega - \phi$  mixing are equal to the ones obtained from the Lagrangian (1). This is also true at higher orders,

but it is necessary to keep in mind that the calculation at higher orders demands taking into account corrections to the  $\phi$  and  $\omega$  wave functions. Certainly, this conclusion is very natural because taking into account all orders of the  $\omega - \phi$  mixing involves nothing more than the diagonalization of the Lagrangian (2) which returns us to the Lagrangian (1). Summing up we conclude that Ref. [1] did not solve the problem of the radiative  $\phi \rightarrow \gamma\pi^0$  decay.

## REFERENCES

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